Newton's Law of Gravitation

Every particle attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$G: 6.67 \times 10^{-11} Nm^2kg^{-2}$$

$$Unit\ of\ F_g: N$$

Gravitational force is always attractive.

$\boldsymbol{F = mg}$ ↔

Gravitation Field

A region of space where a mass will experience a gravitation force.

Gravitational field strength g is defined as the gravitational force per unit mass acting on a small mass placed at that point.

$$g = \frac{F_g}{m} = G\frac{M}{r^2}$$

Factors affecting measurements of g

- Earth's density is not uniform
- Earth is not a sphere but bulging at the equator
- Rotation of Earth at the equator has centripetal force.

↕ $\boldsymbol{F = -\frac{dU}{dr}}$

↕ $\boldsymbol{g = -\frac{d\phi}{dr}}$

Gravitational Potential Energy

Work done by an external force in bringing the mass from infinity to that point in a gravitational field without a change in kinetic energy.

$$U = -G\frac{m_1 m_2}{r}$$

U is always negative. Since the gravitational force is attractive, work must be done by external force to bring it to infinity. As the infinity is taken to be zero (reference point), any other point in the G field will have less GPE.

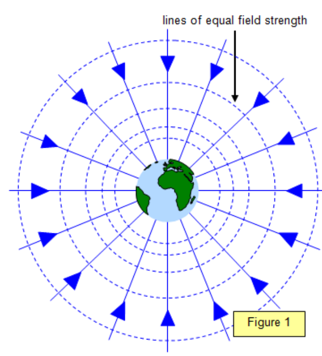
$\boldsymbol{\phi = \frac{U}{m}}$ ↔

Gravitational Potential

Work done per unit mass by an external force in bringing a small mass from infinity to a point in a gravitational field without a change in kinetic energy.

$$\phi = \frac{U}{M} = -G\frac{M}{r}$$

ϕ is always negative. Since the gravitational potential at infinity is taken to be zero, gravitational force is always attractive, and the external force is opposite to the direction of displacement.



--------- Equipotential Lines

⟵ Gravitational Field Lines

Geostationary Orbit: above a fixed point on Earth, Satellite in orbit appears stationary to an observer on Earth.

- Period = 24 hours
- Moving along the equatorial plane.
- Moving from west to east.

Motion and g-field:

Escape Velocity

$$\frac{1}{2}mv_{esc}{}^2 = G\frac{M_E m}{r} \rightarrow v_{esc} = \sqrt{\frac{2GM_E}{r}}$$

Orbiting Satellite

$$G\frac{M_E m}{r^2} = m\frac{v^2}{r} \rightarrow v = \sqrt{\frac{GM_E}{r}}$$

Or

$$G\frac{M_E m}{r^2} = mr\omega^2 \rightarrow T^2 = \frac{4\pi}{GM_E}R^3 \rightarrow T^2 \propto R^3$$

Total Mechanical Energy of Satellite

$$TE = KE + GPE = \frac{1}{2}mv^2 - G\frac{M_E m}{r} = -G\frac{M_E m}{2r}$$