Terminology and Formulas

- For uniform circular motion, there is no work done by the centripetal force since the direction of the force is always perpendicular to the direction of displacement.
- Angular displacement θ (rad): angle swept from a reference position.
- Period T (s): time taken for one complete revolution.
- Angular velocity ω (rad s^{-1}): rate of change of angular displacement with respect to time.
- Linear/Tangential speed v

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$v = r\omega$$

$$a_c = \frac{v^2}{r} = r\omega^2$$

Uniform Circular Motion

- For uniform circular motion, there is no work done by the centripetal force since the direction of the force is always perpendicular to the direction of displacement.
- For uniform circular motion, ω and r are constant, v and a are not because of direction is constantly changing.
- The resultant force causing the circular motion is called the centripetal force. This force is **NOT DRAWN** in a free-body diagram. "The resultant force **provides** the required centripetal force."

$$F_c = \sum F = ma_c = m\frac{v^2}{r} = mr\omega^2$$

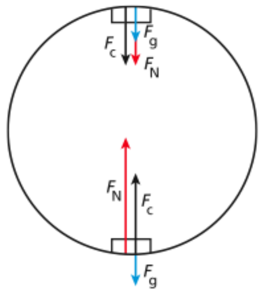
Vertical Circular Motion

- Most examples of vertical circular motion are non-uniform, since the speed and ω are not constant. (Rollercoaster)
- Uniform Circular motion are usually forced to rotate at a constant ω. (Ferris Wheel)

Using Newton's Laws: to explain why an object moving with constant speed in a circle experiences a resultant force towards the centre of the circle.

- Since object experiences a constant change in direction of motion, by N1L, there must be a resultant force on it.
- Given that the tangential speed remains constant by N2L, there must not be any component of force in the tangential direction.
- Hence resultant force must act perpendicular to the velocity, in the radial direction, towards the centre of the circle.

Rollercoaster Example:



__Top of the Circular Motion__

$$F_C = ma_c$$

$$N + mg = ma_c$$

$$N = m(a_c - g)$$

__To Find Speed at the Top__

V is minimum when N = 0.

$$mg = ma_c$$

$$mg = m\frac{{v_{min}}^2}{r}$$

$$v_{min} = \sqrt{gr}$$

__Bottom of the Circular Motion__

$$F_C = ma_c$$

$$N - mg = ma_c$$

$$N = m(a_c + g)$$

Normal Contact Force, N decreases as the object goes up, and is minimum at the top of the circular motion, and maximum at the bottom.

Speed of the rollercoaster also decreases as it gains height. There is a minimum velocity at the top so that the rollercoaster can just remain in contact with the track and complete the circular motion.