# Gravitational Fields

### Gravitational Force *F*

Newton's law of gravitation states that the force of attraction between two point masses is proportional to the product of their masses and inversely proportional to the square of their separation

### Gravitational Field strength *g*

Gravitational field strength at a point is defined as the gravitational force per unit mass acting on a small test mass placed at that point

$$F = \frac{GMm}{r^2} \leftarrow F = -\frac{dU}{dr} \text{ — } U = -\frac{GMm}{r}$$

$$F = mg \qquad U = m\phi$$

$$g = \frac{GM}{r^2} \leftarrow g = -\frac{d\phi}{dr} \text{ — } \phi = -\frac{GM}{r}$$

### Gravitational Potential Energy *U*

Work done by an external force in bringing mass from infinity to point in gravitational field without a change in kinetic energy.

### Gravitational Potential $\phi$

WD per unit mass by an external force in bringing a small test mass from infinity to point in gravitational field without a change in kinetic energy.

Note: Gravitational potential (and energy) is negative because gravitational force is an attractive force and potential is taken to be zero (maximum) at infinity.

### Minimum Escape Speed

(straight upwards from surface of Earth)

By Conservation of Energy, the total energy ($E_K + E_P$) at infinity is greater or equal to zero,

$$\frac{1}{2}mv^2 + \left(-\frac{GM_E m}{R_E}\right) \geq 0$$

$$v \geq \sqrt{\frac{2GM_E}{R_E}}$$

### Orbits

When a satellite (m) orbits around a planet (M), the centripetal force is provided by the gravitational force.

$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$

Kepler's 3rd law:

$$\frac{GMm}{r^2} = mr\omega^2$$

so $T^2 \propto r^3$

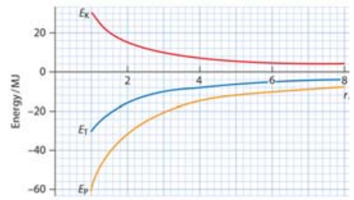
### Energy in Orbits

Starting from $\frac{GMm}{r^2} = m\frac{v^2}{r}$

$$E_K = \frac{GMm}{2r}$$

$$E_P = -\frac{GMm}{r}$$

$$E_{Total} = E_K + E_P = -\frac{GMm}{2r}$$




### Geostationary Satellite

A geostationary satellite is a satellite in a geostationary orbit which allows it to remain stationary relative to an observer on Earth.

Conditions:

1. The period of the satellite orbit is 24 hours
2. The satellite is directly above the equator.
3. The satellite is orbiting in the same direction as the Earth's rotation, west to east.